Chapter 5 TPS4e

Online MC Practice Quiz KEY
Chapter 5: Probability: What Are The Chances?

1. Research on eating habits of families in a large city produced the following probabilities if a randomly selected household was asked “How often during the week do you have a vegetarian (meatless) main dish at dinnertime?”

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three or more times a week</td>
<td>0.06</td>
</tr>
<tr>
<td>Twice a week</td>
<td>0.10</td>
</tr>
<tr>
<td>Once a week</td>
<td>0.49</td>
</tr>
<tr>
<td>Never</td>
<td>?</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected household never has a vegetarian main dish at dinnertime?

A. 0.65

AR. Incorrect. The missing probability must satisfy the fundamental rule that the probabilities of all outcomes in the sample space add to 1. The answer will result in an illegitimate distribution.

B. 0

BR. Incorrect. The event “never serves a vegetarian main dish” should not be confused with the “impossible event” that can never occur. Choosing 0 here will result in an illegitimate distribution.

*C. 0.35

CR. Correct. The missing probability must satisfy the fundamental rule that the probabilities of all outcomes in the sample space add to 1. Thus the probability must be

\[ 1 - (0.06 + 0.10 + 0.49) = 0.35. \]

2. A game consists of drawing three cards at random from a deck of 52 playing cards. You win 3 points for each red card that is drawn. It costs 2 points each time you play. For one play of this game, the sample space \( S \) for the net number of points you gain (after deducting the cost of play) is

*A. \( S = \{-2, 1, 4, 7\} \)

AR. Correct. The possible numbers of red cards that could be drawn are 0, 1, 2, and 3. The possible points won as a result of drawing these numbers of red cards are 0, 3, 6, and 9. Subtracting the cost of play, 2 points, from each of these values yields the sample space you selected.

B. \( S = \{0, 1, 2, 3\} \)

BR. Incorrect. You have chosen values corresponding to the possible numbers of red cards that could be drawn. These are not the same as the amounts won or the net profits.

C. \( S = \{0, 3, 6, 9\} \)

CR. Incorrect. You have chosen values corresponding to the possible total points won. You have neglected to deduct the cost of play.
3. The probability of a randomly selected person being left-handed is \( \frac{1}{7} \). Which one of the following best describes what this means?

*A. If a very large number of people are selected, the proportion of left-handed people will be very close to \( \frac{1}{7} \).

AR. Correct. For a very large number of people, we can apply the law of large numbers and say that the proportion of times a left-handed person is selected will be very close to the actual probability of getting such a person.

B. For every 700,000 people selected, 100,000 will be left-handed.

BR. Incorrect. Saying that the probability of an event is \( \frac{1}{7} \) does not mean that you are certain to get 1 such event out of every 7 chance results. It merely means that in the long run we expect to get this outcome \( \frac{1}{7} \) of the time.

C. If we get 4 left-handed people in 4 consecutive random selections, the probability that the next person is left-handed is substantially lower than \( \frac{1}{7} \).

CR. Incorrect. Random selections are independent (assuming the population is very large). So while we expect the long-run proportion of lefties to be \( \frac{1}{7} \), the probability of choosing a left-handed person is not influenced by previous outcomes. Short-run proportions can depart considerably from \( \frac{1}{7} \).

4. In the wild, 400 randomly selected blooming azalea plants are observed and classified according to flower petal color (white, pink, or orange) and whether or not they have a fragrance. The table gives the results.

<table>
<thead>
<tr>
<th>Fragrance</th>
<th>White</th>
<th>Pink</th>
<th>Orange</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22</td>
<td>104</td>
<td>140</td>
<td>266</td>
</tr>
<tr>
<td>No Fragrance</td>
<td>98</td>
<td>16</td>
<td>20</td>
<td>134</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td>120</td>
<td>120</td>
<td>160</td>
<td>400</td>
</tr>
</tbody>
</table>

If a single azalea plant is selected at random, which one of the following is the probability that it has pink flower petals or no fragrance?

A. 0.04.

AR. Incorrect. This is the probability of a plant having pink flower petals and no fragrance.

B. 0.635.

BR. Incorrect. The events “pink petals” and “no fragrance” are not disjoint. You cannot find the probability that one or the other of the events occurs simply by adding their probabilities.

*C. 0.595.

CR. Correct. We must use the general addition rule here, since the events “pink petals” and “no fragrance” are not disjoint. The probability is \( \frac{120}{400} + \frac{134}{400} - \frac{16}{400} = \frac{238}{400} = 0.595 \).
5. In the wild, 400 randomly selected blooming azalea plants are observed and classified according to flower petal color (white, pink, or orange) and whether or not they have a fragrance. The table gives the results.

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If a single azalea plant is selected at random and found to be orange, what is the probability that it has no fragrance?
A. 0.05.
AR. Incorrect. This is the probability that the plant is orange and has no fragrance.
*B. 0.125.
BR. Correct. Since we know the plant is orange, we determine what proportion of orange plants have no fragrance: \( \frac{20}{160} = 0.125 \).

C. 0.149.
CR. Incorrect. This is the probability that the plant is orange, given that it has no fragrance. Not quite what we want!

6. In the wild, 400 randomly selected blooming azalea plants are observed and classified according to flower petal color (white, pink, or orange) and whether or not they have a fragrance. The table gives the results.

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Suppose a single azalea plant is chosen at random. Which of the following expressions establishes that the events “Fragrance” and “Pink” are independent?
A. \( \frac{104}{400} \neq 0 \).
AR. Incorrect. This demonstrates that \( P(\text{Fragrance} \cap \text{Pink}) \neq 0 \), which shows that these two events are not mutually exclusive.
B. \( \frac{104}{120} \neq \frac{104}{266} \).
BR. Incorrect. This demonstrates that \( P(\text{Fragrance} | \text{Pink}) \neq P(\text{Pink} | \text{Fragrance}) \). This inequality does not establish independence.
*\( \frac{104}{120} \neq \frac{266}{400} \).
CR. Correct. This demonstrates that \( P(\text{Fragrance} | \text{Pink}) \neq P(\text{Fragrance}) \). In other words, the probability of randomly selecting a plant with fragrance is not changed by knowing it is pink.
7. Suppose we toss a fair penny and a fair nickel. Let A be the event that the penny lands heads and B be the event that the nickel lands tails. Which one of the following is true about events A and B?

A. A and B are disjoint.
AR. Incorrect. Keep in mind that the events A and B are associated with different coins. If these events were disjoint, they could not happen at the same time. But if you toss a penny and a nickel, it’s certainly possible to get heads on the penny and tails on the nickel. The probability of such an event in this case is \((1/2)(1/2) = 1/4\). Because A and B can occur together, they cannot be disjoint.

B. A and B are complements.
BR. Incorrect. Keep in mind that the events A and B are associated with different coins. The complement of an event occurs when the event does not occur. The probability of the complement of A is the event that the penny shows tails. It has nothing to do with the nickel.

*C. A and B are independent.
CR. Correct. Since the tosses of the penny and the nickel do not influence each other, any event concerning the outcome for the penny is independent of any event concerning the outcome for the nickel. The event \(A = \text{“the penny shows heads”}\) is therefore independent of the event \(B = \text{“the nickel shows tails”}\).

8. You want to perform a simulation to estimate the probability of getting at least one run of 3 heads in a row in 10 flips of a fair coin. Which of the following describes a correct simulation for estimating this probability?

A. Assign the numbers 0 through 4 to “heads” and 5 through 9 to “tails.” Read 500 1-digit numbers from a random digits table and count how many times you get runs of three or more heads in a row. Divide the count by 500.
AR. Incorrect. You are trying to determine what proportion of the time something happens in 10 flips of a coin. This simulation does not involve repeated sets of 10 flips.

*B. Assign the numbers 0 through 4 to “heads” and 5 through 9 to “tails.” Read 10 one-digit numbers from a random digits table, and count this simulation as a “success” if there is at least one run of 3 or more heads. Repeat this 500 times and divide the total number of “successes” by 500.
BR. Correct. This simulation will estimate the proportion of times that 10 flips of a coin will produce a run of at least 3 heads. Each random number selected represents a single coin flip, and ten of these “flips” constitute a single simulation.

C. Flip a coin 10 times. Count the 10 flips as a “success” if there are at least 3 heads in that set of 10 flips. Repeat this 500 times and divide the total number of “successes” by 500.
CR. Incorrect. This simulation estimates the probability of getting 3 or more heads in 10 flips, not the probability of getting 3 or more heads in a row.
9. In a large city, 82% of residents own a cell phone. Suppose that we randomly select three city residents. What is the probability that at least one of the three residents does not own a cell phone? [The city is large enough so that we can assume independence].

A. 0.994
AR. Incorrect. You have calculated the probability that at least one of the three residents has a cell phone.
*B. 0.449
BR. Correct. The complement of the event “at least one of the three residents does not own a cell phone” is the event “all three residents do have cell phone.” By the multiplication rule for independent events, the probability of this event is \((0.82)(0.82)(0.82) = (0.82)^3 = 0.551\). The desired probability is, therefore, \(1 - 0.551 = 0.449\).
C. 0.006
CR. Incorrect. You have calculated the probability that none of the three residents has a cell phone.

10. A jar contains 10 red marbles and 15 blue marbles. If you randomly draw two marbles from the jar (without replacement), what is the probability that they are the same color?

*A. 0.5
AR. Correct.

\[
P(\text{Two reds or two blues}) = P[(R \cap R) \cup (B \cap B)] = \frac{10}{25} \cdot \frac{9}{24} + \frac{15}{25} \cdot \frac{14}{24} = \frac{90}{600} + \frac{210}{600} = 0.5
\]
B. 0.15
BR. Incorrect. This is only the probability of getting two red marbles.
C. 0.52
CR. Incorrect. You have not taken into account that the marbles were drawn without replacement.

11. A blood test for a certain disease has a false positive rate of 0.01 and a false negative rate of 0.05. (Recall that “false positive” means the test returns a positive result when the subject does not have the disease). Suppose that 2% of a certain population has the disease. If a random individual from this population tests positive, what is the probability that this person actually has the disease?

A. 0.019
AR. Incorrect. This is the probability that a random person has the disease and tests positive.
B. 0.0288
BR. Incorrect. This is the probability that a random person tests positive.
*C. 0.6597
CR. Correct. Using the conditional probability formula, with \(D = \text{Person has the disease}\) and \(\text{“+”} = \text{person tests positive}\),

\[
P(D | +) = \frac{P(D \cap +)}{P(+)} = \frac{(0.02)(0.95)}{(0.02)(0.95) + (0.98)(0.01)} = 0.6597.
\]
12. A survey of high school students finds that 80% of them get news on current events from the internet, 25% of them get news from television, and 15% use both sources. Which of the following is an accurate Venn diagram of this information? [Let “I” = Get news from internet and “T” = Get news from Television.]

A. Incorrect. Note that all of the numbers inside a given circle should add up to the total proportion for that circle. For example, the total number of students who get their news from the internet should be the sum of all values in the “I” circle.

B. Correct. Note that if 80% use the internet but 15% use both, we know that 65% use the internet but not television.

C. Incorrect. Note that when we say “80% use the internet” we are including those who use both sources.